Complex Compensation of Coupled Line Structures in Inhomogeneous Media

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***Abstract*— This paper presents a novel method for the compensation of unequal phase velocities of coupled line structures in inhomogeneous media. By means of a rigorous scattering parameter analysis, ideal values for compensating impedances, connected in series to one or several ports of the coupler, are found. It is shown that, by properly choosing the values of the impedances' real and imaginary part, perfect isolation can theoretically be obtaind at any frequency. Simulation and measurements are compared to previously published results, demonstrating the advantage of this novel approach.**

I. Introduction

Coupled line structures are widely used in microwave circuits, serving as directional couplers or being key components in filters, baluns, matching networks or combiners. If realized in an inhomogeneous medium as encountered in microstrips, the effective permittivities of the coupled lines' even and odd mode differ. As a consequence, the two modes exhibit different phase velocities which leads to poor directivity for couplers, imbalance in case of marchand baluns and spurious modes at the first harmonic for coupled line filters. Among the known methods for compensating unequal phase velocities one can distinguish between two categories. The first type aims to equalize the effective permittivities of odd and even mode by altering their electromagnetic fields. This can be done by placing dielectric layers above the coupling structure [1], [2], employing anisotropic substrates [3], realizing quasi suspended substrate structures [4], or etching apertures into the ground plane [5], [6]. The main disadvantage of theses methods is the lack of close-form design equations. Therefore, numerous iterations during the design process have to be carried out.

The second class of compensation methods relies on externally connected reactive components. These can be placed between the coupled lines [7]–[9], shunted with the coupled lines [10], or connected in series to one ore several ports [11], [12]. The possibility of compensating unequal phase velocities by connecting inductive elements in series to the ports of a coupler was originally proposed in [13]. The first analytical approach for this method was recently published in [11], [12], where the coupler including the compensating inductance L was analyzed via the Z-matrix. Further, an optimal value for L was sought

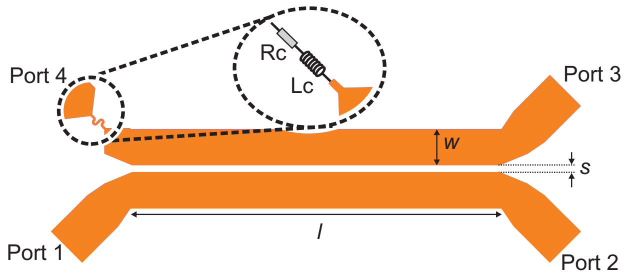

Fig. 1. Microstrip coupler with complex compensation at one port.

for under the condition of minimum leakage S_{31}. For the singly compensated case (inductive element at port 4), a minor improvement in isolation of 8 dB was obtained. In the doubly compensated case (inductive elements at port 2 and port 4) the isolation exhibits a peak at 25 dB above the uncompensated case. Hence, it was concluded that the doubly compensated case is superior in terms of isolation and directivity enhancement. It was further noted, that the frequency of maximum isolation was shifted with respect to the center frequency of the coupler.

In the following we analyze a coupler featuring compensating elements by means of the scattering parameters. In contrast to [11], [12], our analysis does not only yield a reactance as the compensating element, but a complex impedance (see Fig. 1). The so found values of the real and imaginary part lead to perfect isolation peaks for both the one-element and the two-element compensation. Further it will be shown that the isolation peak can be obtained at any frequency.

II. Theory and Analysis

The underlying thought of our approach is that the compensating element creates an internal reflection which - with proper amplitude and phase - compensates for the leaking S_{31} of the uncompensated coupler. An analytical expression for the optimal reflection coefficient is derived and the compensating impedance is calculated therefrom. The analysis will be carried out for the one-element and for the two-element compensation. The formulas for the scattering parameters of coupled lines in inhomogeneous media are taken from [14].

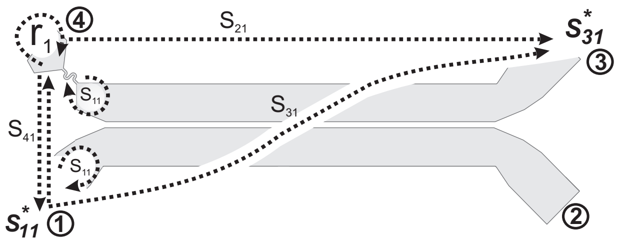

Fig. 2. Flow graph analysis for a coupler with one compensating element.

A. One-element compensation

We consider a signal incident at port 1 of an uncompensated coupler, as shown in Fig. 2. A leaking S_{31} will be observed at port 3 in case of an inhomogeneous medium. By placing a compensating element at port 4, a part of the signal S_{41} is reflected and will superpose with the leaking S_{31}. The considered uncompensated coupler is symmetric, thus $S_{ii} = S_{11}$ $(i = 1, 2, 3, 4)$, $S_{34} = S_{21}$ and $S_{32} = S_{41}$. The resulting S^*_{31} can be written as

$$S^*_{31} = S_{31} + \frac{S_{41} \cdot r_1 \cdot S_{21}}{1 - S_{11} \cdot r_1}. \tag{1}$$

With the isolation condition $S^*_{31} = 0$ one can solve for the optimal reflection coefficient at port 4:

$$r_{1opt} = -\frac{S_{31}}{S_{41} \cdot S_{21} - S_{11} \cdot S_{31}}. \tag{2}$$

This reflection can be created by an impedance, which is calculated from

$$Z_c = Z_0 \cdot \left\{ \frac{1 + r_{1\,opt}}{1 - r_{1\,opt}} \right\}, \tag{3}$$

where $Z_0 = \sqrt{Z_{even} \cdot Z_{odd}}$ is the reference impedance and $Z_c = R + jX$ is the compensating impedance, seen at port 4 from inside the coupler. Inserting (2) into (3) the optimal value of the impedance can be derived:

$$Z_c = Z_0 \cdot \left\{ \frac{S_{41} \cdot S_{21} - S_{31} \cdot (1 + S_{11})}{S_{41} \cdot S_{21} + S_{31} \cdot (1 - S_{11})} \right\}. \tag{4}$$

Using a series connection, the values of the resistance and the reactance are obtained from

$$R = \mathrm{Re}\,\{Z_c\}\,, \tag{5a}$$

$$X = \mathrm{Im}\,\{Z_c\}\,. \tag{5b}$$

The final formula for the compensating impedance is obtained by inserting the equations of the scattering parameter from [14] into (4). Real parts differing from Z_0, the characteristic impedance of the connected transmission line, can be realized by means of a $\lambda/4-$transformer. Here, λ is the wavelength on the transmission line at the design frequency. It can be shown that the values of the compensating impedances are independent of both the coupler's center frequency and the absolute values of the effective permittivities. On the other hand, they are strongly dependent on the coupling coefficient $c = 20 \cdot log_{10}((Z_{even} - Z_{odd})/(Z_{even} + Z_{odd}))$ as well as on the ratio of the even and odd mode permittivity $k = \epsilon_{even}/\epsilon_{odd}$, as it is depicted in Fig. 3. For an ideal coupler with equal effective permitivities ($k = 1$), the required compensating impedance becomes Z_0. For $k > 1$, as it is the case for microstrip couplers, the compensating reactance is positive and can be realized with a series inductance $L = X/(2\pi f_0)$, f_0 being the coupler's center frequency. For $k < 1$, as in suspended substrate technology, the reactance is negative and can be realized with a series capacitance $C = -1/(2\pi f_o X)$. The real part is approximately Z_0 in case of strong coupling ($c > -6$ dB). For weaker coupling the real part decreases significantly with growing inhomogeneity. As depicted in Fig. 3b, a 20 dB coupler with $k \geq 1.3$ would require a compensating impedance with $R \leq 0$, which is not realizable passively. Under these circumstances the coupler is thus not compensatable by means of a one-element compensation. So far, the compensation was realized at the coupler's center frequency. Fig. 4 shows, that by properly choosing R and X, the compensation can be obtained at any frequency. The simulation was carried out for a 10 dB coupler with $k = 1.25$ and $Z_0 = 50\,\Omega$. The frequency is normalized to an arbitrary center frequency f_0. It can be observed that for isolation peaks below the center frequency, both the optimal R and L become larger. Setting the resistance to Z_0, the isolation peak is shifted to around $0.85\,f_o$.

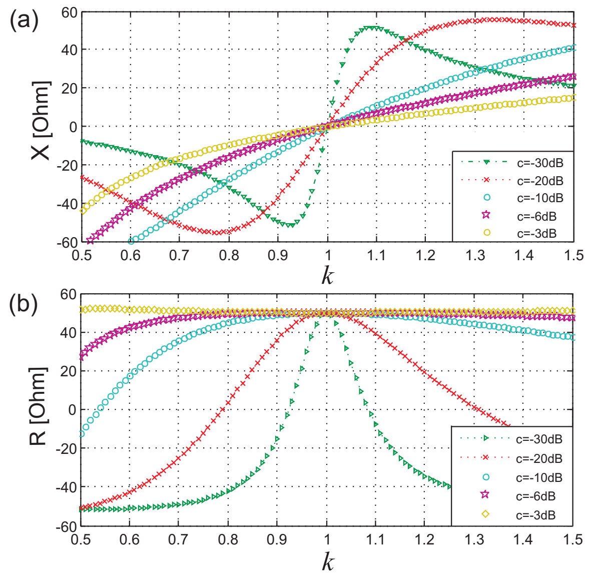

Fig. 3. The compensating impedance Z_c for $Z_0 = 50\,\Omega$ as a function of the permittivity ratio $k = \epsilon_{even}/\epsilon_{odd}$ for different couplings: (a) reactance, (b) resistance.

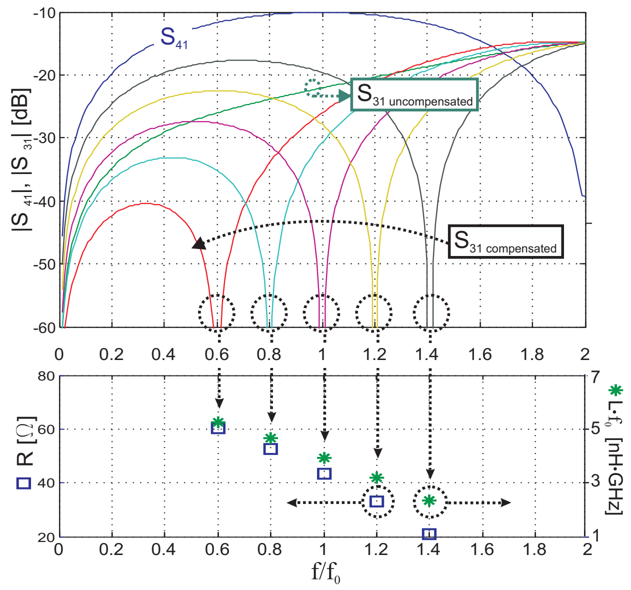

Fig. 4. Compensation of a 10 dB coupler at different frequencies by means of the one-element compensation. The corresponding values of the compensating impedances are given below.

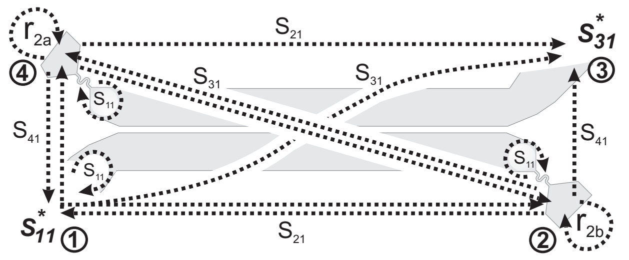

Fig. 5. Flow graph analysis for a coupler with two compensating element.

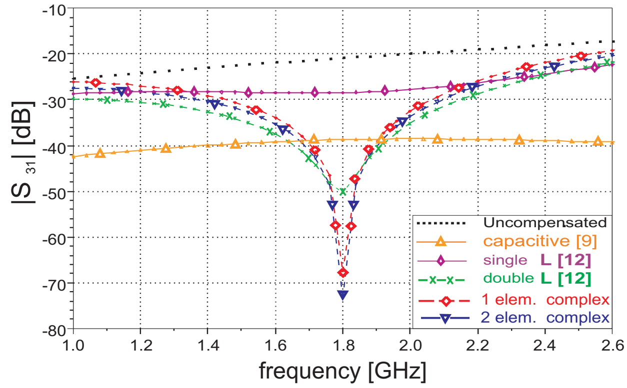

Fig. 6. Simulated results of the magnitude S_{31} for the uncompensated case($\cdots$) and various compensated cases: capacitive($-\Delta-$) [9], singly-($-\diamond-$) and doubly inductive($-X-X-$) [12], and one element-($-\diamond-\diamond-$) and two element complex ($-\nabla-$) compensation (this paper).

B. Two-element compensation

In case of a two-element compensation, several paths and loops have to be taken into account during the flow-graph analysis, as it is depicted in Fig. 5. For the sake of simplicity we take the same compensating impedances at port 2 and port 4, thus obtaining equal reflections $r_{2a} = r_{2b} = r_2$. The isolation condition $S^*_{31} = 0$ can be solved for the optimal reflection coefficient as:

$$r_{2opt} = \frac{-\mathrm{A} \pm \sqrt{\mathrm{A}^2 - 2 \cdot \mathrm{B} \cdot S_{31}}}{\mathrm{B}}, \tag{6}$$

where

$$\mathrm{A} = 2 \cdot (S_{21} \cdot S_{41} - S_{11} \cdot S_{31}), \tag{7a}$$

$$\mathrm{B} = 2 \cdot S_{31} \cdot (1 - 2 \cdot S^2_{31}) - 4 \cdot S_{11} \cdot S_{21} \cdot S_{41}. \tag{7b}$$

In the same manner as in (3), the required impedance can be calculated therefrom. In Fig. 6 the simulated results of the complex one- and two-element compensation are compared to the cases presented in [9] and [12] and to the uncompensated coupler. The simulation are made for a 10 dB coupler on a substrate with thickness $h = 1.52\,mm$ and $\epsilon_r = 6.45$ (*RF60-0600* from *Taconic*). The optimal values of the compensating impedances are found to be $R = 41.5\,\Omega$, $L = 2.39\,nH$ and $R = 47.5\,\Omega$, $L = 1.3\,nH$ for the one element and for the two element compensation, respectively. It is clearly seen, that both approaches lead to the predicted perfect isolation at the center frequency. The optimal values found in [12] are $L_s = 1.24\,nH$ and $L_d = 1.31\,nH$ for the singly and doubly compensated case, respectively. Furthermore, the isolation peaks in [12] were originally observed at a frequency below the center frequency. The overall length of the coupler was then shortened, in order to shift this peak to the center frequency. The capacitive compensation of [9] however, shows an advantage over the other methods regarding the bandwidth. Further, the effect of the compensation impedances on the input reflection at port 1 has been investigated. Fig. 7 shows the input reflection as a function of the coupling for the uncompensated coupler as well as for the two compensated variants. The inhomogeneity was taken as $k = 1.25$. For couplings from $c = -1$ dB to $c = -30$ dB the S_{11} of the uncompensated coupler decreases from -30 dB to around -40 dB. The input reflection of the one-element compensation remains always about 20 dB below. In contrast, the two-element compensation exhibits an input reflection that increases up to -6 dB for a 30 dB coupler. The reason for this tremendous degradation is that a coupler with weak coupling c needs large reflections at port 2 and port 4 for phase compensation. As a consequence, a large part of the incident signal is reflected back to the input port with almost no attenuation because $S_{21} > -1$ dB. With an equally good isolation, but a better input reflection, the one-element compensation method is consequntly the preferred choice among the two variants.

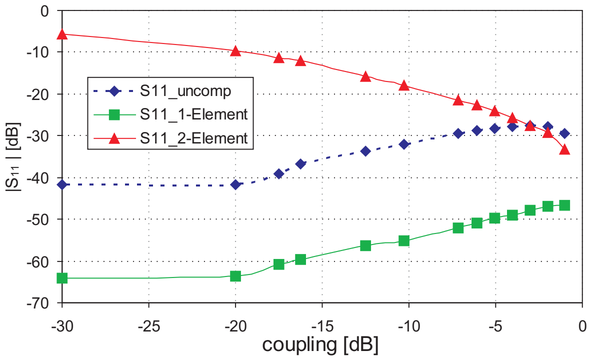

Fig. 7. Input reflection of compensated and uncompensated couplers as a funtion of the coupling coefficient for $k = 1.25$.

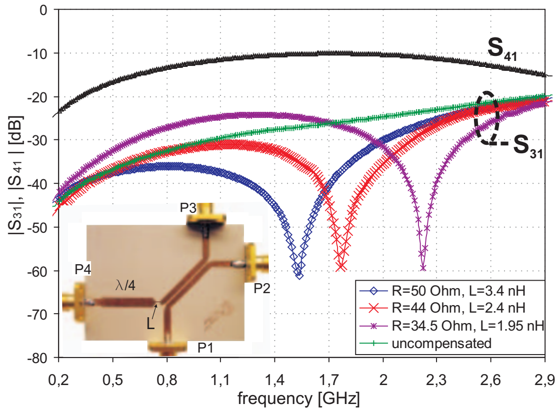

Fig. 8. Measurement results of a 10 dB coupler compensated at various frequencies.

III. Measurement Results

To validate our approach, several 10 dB couplers were fabricated and measured. As a substrate we used *RF60A-600* of *Taconic*, with $\epsilon_r = 6.15$ and a thickness of $h = 1.52$ mm. The couplers were designed for a center frequency of 1.8 GHz and with isolation peaks at 1.5, 1.8 and 2.2 GHz respectively. The results are shown in Fig. 8. The coupler's dimension are $s = 0.35$ mm, $l = 19$ mm and $w = 1.8$ mm (see Fig.1 for the notation). As can be seen in the inset of Fig. 8, a $\lambda/4$-section was used to transform the $50\,\Omega$ reference impedance to the desired resistance of the compensating element. Isolation peaks as high as 60 dB were measured, which corresponds to an improvement of more than 35 dB compared to the uncompensated coupler. The coupler with $R = 50\,\Omega$ has its peak at 1.5 GHz, demonstrating the shift to lower frequencies, when neglecting the real part. The reflection coefficient at port 1 is around -40 dB for all compensated couplers and around -30 dB for the uncompensated one, which confirms the results of the reflection analysis presented in Fig. 7.

IV. Conclusion

We presented a novel phase compensation method for coupled line structures in inhomogeneous media. By means of a rigorous flow-graph analysis, optimal values for the real and imaginary parts of the compensating impedance were obtained. The frequency shift of the isolation peak mentioned in previous publications can be explained with the neglect of the real part. Further, it was shown that the one-element compensation is superior to the two-element compensation, because it does not only lead to a perfect isolation, but also improves the input reflection. To validate the novel approach, we designed and fabricated three 10 dB couplers with isolation peaks at different frequencies. The measurement results agree with the simulation and demonstrate the advantage of our approach compared to known methods.

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